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Gauge constraints and electromagnetic properties of off-shell particles

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Abstract. The consequences of the gauge constraints for off-shellness in the electromagnetic (EM) vertices have been considered, using Compton scattering as an example. We have found that even if the gauge constraint for the 3-point EM Green function allows for off-shell effects in the charge (Dirac) form factor, they vanish in the total Compton scattering amplitude due to the gauge constraint for the 4-point EM Green function only. In addition, the representations of the Compton amplitude in terms of either reducible or irreducible vertices are equivalent for conserved currents.

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1 Introduction

A deviation of the electromagnetic (EM) properties of the off-shell (virtual/bound) particles from their free states, and its possible manifistation in the high-energy hadron/nuclear electrodynamics is an intriguing problem. The EM vertices for off-shell hadrons may have a more complicated structure in comparison with the on-shell ones. For example, the number of the off-shell form factors (FF) in the general $\gamma^{\star}NN$ vertex with one (two)-off-shell nucleon(s) increases from two FF for the free nucleon to 4 (6), and each of these becomes a function of the additional *hadronic* virtualities [1]. So, the question whether it is possible to observe such a “modification” of the nucleon EM properties in the nuclear medium has been discussed for a long time. One of the most common theoretical approach to evaluate the off-shell behaviour of the various amplitudes through the physical observables is a method of dispersion relations. First un-subtractive dispersion relations for the off-shell FF were introduced by Bincer [1]. Subsequently “sidewise” dispersion relations, which connect the structure of the nucleon form factors with the physics of baryon resonances, were widely used [2]. It seemed, that they could allow one to evaluate the off-shell properties through the physically observed nucleon resonances, i.e. observables such as phase shifts in meson-nucleon scattering [3]. Only recently the physical meaning of “sidewise” dispersion relations was reexamined [4]. It was shown that such a connection between the off-shell form factors and the observables in meson-nucleon scattering in reality depends on the definition of the intermediate field operators: their redefinition leaves the scattering amplitude invariant, but changes the behaviour of the off-shell form factors, showing their representation dependence. Nevertheless, up to now speculations about medium modification of the nucleon charge form factor in the nuclear medium and the possibility to “measure” such an “effect” experimentally remain.

It is well known that any off-mass-shell extension of a matrix element is not unique: various representations of the field operators give rise to different off-shell Green functions and off-shell matrix elements, while all of these lead to the same S-matrix. Off-shell properties of Green functions and in particular their dependence on the representation of the Lagrangian have been discussed in the framework of field theory for a long time [5]. Recently the method of effective Lagrangians was applied to real Compton scattering where the representation dependence of off-shellness has been pointed out [6]. The possibility to move off-shell effects from irreducible vertices of the Born current to the regular part of the amplitude was used [6] to obtain a unique definition for polarizabilities in Compton scattering on nucleons. Also in chiral perturbation theory applied to real Compton scattering off the pion, it was shown [7] that off-shell effects depend not only upon the model of Lagrangian, but also on its representation. The possibility to shift any explicit off-shell dependence of the irreducible three-point Green function to the regular part of the amplitude has also been used to derive [8,9] the low energy theorem for the virtual Compton scattering (VCS).

Taking into account that form factor is a *pure individual* "property" of the particle, it is clear that the definition of an *off-shell* FF is directly connected with the possibility to introduce a *one-body off-shell* EM current independently of the full EM reaction amplitude. However, due to requirements of gauge invariance one- and many-body currents cannot be considered separately [10]. Since the one-body off-shell current is not conserved itself, the definition of off-shell FF will not be gauge invariant. Therefore, even starting from the general fundamental requirements of gauge invariance, it is evident that off-shell effects in the $\gamma^{\star}NN$ vertex should be considered in connection with the representation of the full conserved current.

In the space-like region (and also for $q^2 > 4M^2$), where the on-shell $\gamma^{\star}NN$ vertex corresponds to a physical process, an *on-shell* EM FF can be defined in a gauge invariant manner (they are known, or at least, may be measured), while any off-shell effects may be considered as corrections and formally transferred to the non-Born part of the amplitude [8,9,11]. On the other hand, in the so-called "unphysical" region ($4m_e^2 < q^2 < 4M^2$) for any EM process one of the hadrons in the EM vertices is *off-mass-shell* and hence only *off-shell* EM FF exist. However, their definition cannot be gauge invariant and such FF cannot be investigated in practice. That is why in the case of a virtual time-like photon it is important to understand whether such a representation of the amplitude exists, that the off-shell effects enter only in the non-pole transverse (gauge invariant) components of the total conserved current, whereas the pole-type piece contains only "on-shell extrapolated" FF.

In general there are various ways (which lead to the same physical result) to construct a covariant and gauge invariant VCS amplitude [7-9]. Here we will use a field-theoretical approach [10] formulated in terms of the n-point Green functions, and explore the gauge nature of the EM field (associated with the real photon). The Ward-Takahashi identities (WTI), which connect the (n+1)-point EM Green functions (with EM field) and the hadronic n-point Green functions (without EM field), are an important ingredient of the gauge invariant theory in this case.

In this paper we focus on the off-shell effects in the *charge* (Dirac) form factors only, since only they are "affected" by the gauge constraints on the level of 3- and 4-point EM Green functions, i.e. on the "tree"- level already [10]. In the case of *reducible* vertices, for instance, they do not contain any off-shellness at the "photon point" at all due to the gauge constraint for 3-point EM Green functions [1,2,10]. However, in the case of *virtual* photons, or *ir-reducible* vertices, gauge constraint for 3-point EM Green function allows a *hadronic* off-shellness in the charge (Dirac) coupling, while some additional restrictions for off-shellness appear from the gauge constraint for the 4-point EM Green functions. To see this we need not only to introduce an exactly conserved EM current, but it is also important to define its *irregular* and *regular* pieces (pole and contact current) consistently, from the "same principle". The "minimal insertion" of the gauge field into n-point Green functions provides this on the basis of the fundamental requirements only (analogously to QED [12]). On this way the off-shell Born current (irregular part) and contact current (regular part) may be defined from the same off-shell $\gamma^{\star}NN$ vertex for which only the general structure will be assumed.

We note that the present consideration differs from previous studies of the VCS [8-9,11] based upon an *independent* introduction of an explicite "on-shell Born current" and an unknown (in principle) "regular part", transferring all possible off-shell effects to the last one, which was then parametrised in terms of the invariant (un-known) functions - "generalized polarizabilities" - on the basis of the fundamental requirements of covariance, CPT and gauge invariance. Of course, such a consideration allowed to show a representation dependence of the off-shell effects [9]. However, that was not enough to find a complete cancellation of the off-shellness in the charge (Dirac) form factors.

The paper is arranged in a following way. In Section 2 a purely EM process $\gamma + p \Longleftrightarrow \gamma^{\star} + p$ (space- or time-like VCS off a proton) is considered. First, we give the definitions of the various hadrons and EM Green functions (without and with EM field), as well as corresponding WTI to which the last ones should satisfy. Then, using minimal substitution of the gauge field into the corresponding Green functions, the structure of the *off-shell* Born current, the contact current and the total conserved VCS current is obtained on the basis of the same principle, in terms of the *reducible* vertices and *free* Feynman propagators. We show that *off-shell* form factors cannot be defined in a gauge invariant way, even if $\gamma^{\star}NN$ vertex satisfy corresponding WTI, and, as a result, they cannot be investigated in reality.

In Section 3 we deal with the "off-shellness" in the *regular* and *irregular* pieces of the conserved current. In a model independent way we show that off-shell effects, which appear only for the *virtual* photons in the *reducible* $\gamma^{\star}NN$ vertices, may be not only moved from the pole-part of the amplitude to its regular part, but also *cancelled* in the charge operators of the total conserved current, if the pole and contact amplitudes are considered consistently. Additionally, we will see that the form factors $F_{1,2}^{-}$, connected with negative energy states of the virtual proton, will be cancelled in the total conserved current and will not influence the observables.

Section 4 is devoted to an alternative construction of the total conserved current in terms of the *irreducible* vertices and *full* renormalized propagators, again on the basis of the minimal substitution procedure. Another representation of the "minimal" conserved current for VCS is obtained there. The relation between two different representations of the conserved current, based on the *reducible* and *irreducible* $\gamma^{\star}NN$ vertices, is established. In the framework of the "minimal" scheme, using general properties of the mass-operator, it was shown that the "corrections" to the Born and contact currents, stipulated by the self-energy part, have the same absolute value but opposite signs. So, they cancel one another in the total conserved current in a such a model independent way that the amplitudes defined in terms of the *reducible* vertices with *free* Feynman propagators and *irreducible* ones together with *full* renormalized propagators become the same. At last, for the *real* photons the off-shellness in the *irreducible* vertices may be isolated inside the "correction terms" only and, due to their cancellation in the total current, may be not considered at all. This simply corresponds to the transition to the representation of the conserved current in terms of the *reducible* vertices which are free from the off-shellness due to the gauge constraint.

Section V is devoted to an application of the gauge invariant definition of the "on-shell extended" form factors in the time-like *unphysical* region, which is only possible due to special representation of the total amplitude, and is based on the cancellation of the off-shellnes in the conserved current. The sensitivity of the e^+e^-- asymmetry for the di-lepton production off the proton in a "special" kinematics to such a "gauge invariant" form factors is considered. In Section VI we discuss the results and give the conclusions. Some details and intermediate transformations are given in the Appendices.

2 Structure of the VCS amplitude

In this section the structure of the gauge invariant nucleon current for VCS off a proton with accounting for the off-shell effects in the EM vertices will be considered. For a Dirac coupling (in relation to the *real photon*) or scalar particles a consistent definition of the regular and irregular pieces of the total conserved current, based on the gauge nature of EM field, is possible without a special model for the off-shell behaviour of the FF. To derive this, let us introduce first strong and EM Green's functions, as well as the corresponding WTI to which they should satisfy.

2.1 Preliminaries

The two-point Green function of the proton, as well as the electromagnetic three- and four-point Green functions are defined as

$$S(x,y) = -i < 0|T\{\psi(x)\bar{\psi}(y)\}|0>,$$

$$G_\mu(x,y,z) =< 0|T\{\psi(x)\bar{\psi}(y)j_\mu(z)\}|0>,$$

$$G_{\mu\nu}(x,y,z,r) =< 0|T\{\psi(x)\bar{\psi}(y)j_\mu(z)j_\nu(r)\}|0>,$$

where $j_\mu(r) = -\nabla^2 A_\mu(r)$ is the EM current operator, $\psi(x)$ denotes an interpolating field operator of the proton, and A_μ is EM field satisfying Lorentz condition $\nabla_\mu A_\mu(r) = 0$; T denotes the time-ordered product.

Using translation invariance the corresponding momentum-space Green functions are

$$(2\pi)^4\delta^4(p-p')S(p) = \int d^4x d^4y e^{i(px-p'y)} S(x,y),$$

$$(2\pi)^4\delta^4(p-p'-q')G_\mu(p,p') = \int d^4x d^4y d^4r e^{i(px-p'y-q'r)} G_\mu(x,y,r),$$

$$(2\pi)^4\delta^4(p+q-p'-q')G_\mu(P,q,q') = \int d^4x d^4y d^4r d^4z e^{i(px+qz-p'y-q'r)} G_\mu(x,y,z,r),$$

where $P = p + p'$, and q, q' are the photon momenta assosiated with currents j_μ and j_ν.

The truncated three- and four-point electromagnetic Green functions, or γNN and $\gamma NN\gamma$ vertices are obtained by "amputation" of the external proton lines:

$$\begin{Bmatrix} \Gamma_\mu(p,p') \\ \Gamma_{\mu\nu}(P,q,q') \end{Bmatrix} = S^{-1}(p) \begin{Bmatrix} G_\mu(p,p') \\ G_{\mu\nu}(P,q,q') \end{Bmatrix} S^{-1}(p')$$

To be more concrete from now on we will distinguish the half-off-shell *reducible* γNN vertex ($\Gamma_\mu(p,p')$) and the *irreducible* one ($\Gamma^{\rm ir}_\mu(p,p')$)

$$S(p)\Gamma^{\rm ir}_\mu(p,p')S(p') = S_0(p)\Gamma_\mu(p,p')S_0(p'), \qquad (1)$$

where $S(p)$ and $S_0(p)$ is the full renormalized and free Feynman propagators.

In the case of *half-off-shell* γNN irreducible or reducible vertices (with real or virtual photons) the corresponding WTI have the same form (corresponding spinors for on-shell particle is implied here, see Appendix A):

$$q_\mu\Gamma^{\rm ir}_\mu(p,p') = e\{S^{-1}(p) - S^{-1}(p')\}, \qquad (2)$$

$$q_\mu\Gamma_\mu(p,p') = e\{S_0^{-1}(p) - S_0^{-1}(p')\}. \qquad (3)$$

However, (2) is more general and correct for full off-shell vertex.

2.2 Conserved current in terms of reducible vertices

A conserved current for a purely EM reaction $\gamma + p \Longleftrightarrow \gamma^\star + p$ can be obtained using minimal substitution of the EM field into the three-point Green function [10], corresponding to the $\gamma^\star NN$ vertex (with virtual photon). In general this procedure may be carried out in two different ways, namely on either the right-hand or the left-hand side of (1). In the first case the theory will be formulated in terms of the *reducible* vertices and free Feynman propagators, while the second case will lead to the formulation in terms of *irreducible* vertices and full renormalized propagators. Even from the beginning it is evident that the same results for physical observables should be obtained, since a gauge (massless) field is "inserted" into the different sides of identity. In this section we start with the right-hand side of (1) and therefore we concentrate on the off-shellness in the *reducible* vertices. (The latter occurs only for virtual photons and does not exist at the "photon point".)

The minimal insertion of the second (real) photon into the external lines of the right-hand side of (1) generates [10] the one-body *off-shell* Born current (see Appendix B)

$$J^{B-off}_{\mu\nu} = e\{\Gamma_\mu(p, p'+q')S_0(p'+q')\gamma_\nu + \gamma_\nu S_0(p-q')\Gamma_\mu(p-q', p')\}. \quad (4)$$

Here $p'(p)$ is initial (final) momentum of the nucleon, $q(q')$ is the virtual (real) photon momentum: $p+q=p'+q'$; e is a charge, and γ_ν is a 4×4 Dirac matrix.

The minimal photon insertion inside the $\gamma^\star NN$ vertex gives rise to the contact ("many-body") current [10], containing only one-body irreducible graphs (diagrams which cannot be disconnected by cutting only any single-particle internal lines). Note, that the requirement of gauge invariance itself does not fix the contact current completely, since it still depends on the "trajectory". The additional assumption of the *minimal* insertion of the EM field, corresponding to the "minimal (linear) trajectory", makes its definition unique [10]

$$J^C_{\mu\nu} = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p, p'+\lambda q') - \Gamma_\mu(p-\lambda q', p')\}. \quad (5)$$

It is easy to check that the derivative of $J^C_{\mu\nu}$ cancels corresponding derivative of $J^{B-off}_{\mu\nu}$ hence the total nucleon current

$$J^N_{\mu\nu} = J^{B-off}_{\mu\nu} + J^C_{\mu\nu} \quad (6)$$

is conserved for any off-shell $\gamma^\star NN$ vertex, independent of its explicit form (initial and final spinors are implied here)

$$q_\mu J^N_{\mu\nu} = J^N_{\mu\nu}q'_\nu = q_\mu(J^{B-off}_{\mu\nu} + J^C_{\mu\nu}) = (J^{B-off}_{\mu\nu} + J^C_{\mu\nu})q'_\nu = 0.$$

We note that $J^{B-off}_{\mu\nu}$ and $J^C_{\mu\nu}$ are not gauge invariant separately

$$J^{B-off}_{\mu\nu}q'_\nu = -J^C_{\mu\nu}q'_\nu = e\{\Gamma_\mu(p, p'+q') - \Gamma_\mu(p-q', p')\} \neq 0,$$

although $q_\mu J^{B-off}_{\mu\nu} = q_\mu J^C_{\mu\nu} = 0$. Only the *on-shell* Born current, $J^{B-on}_{\mu\nu}$, is conserved [8], since the corresponding vertex depends only upon the photon momentum: $\Gamma^{on}_\mu(p, p') \equiv \Gamma_\mu(p-p')$.

The crossing properties of the total current (6) are considered in Appendix C.

2.3 Gauge dependence of the definition of off-shell EM FF

Since the one-body off-shell current is not conserved (see above), the definition of the off-shell FF is not gauge invariant in spite of the fact that the half-off-shell $\gamma^\star NN$ vertex satisfies the WTI for the three-point Green function. Indeed, if one of the nucleons is off-mass-shell, the vertex $\gamma^\star NN$ is only a part of the one-body reducible graph (nucleon-pole diagram). The corresponding one-body off-shell current cannot satisfy the next order WTI (for the Green function of the considered process, for example, the four-point one) itself, without a contribution of the "many-body" (contact) current, i.e. the regular piece of the amplitude. Therefore, the off-shell $\gamma^\star NN$ vertex cannot be considered independent of the full conserved current and the definition of the off-shell FF cannot be gauge invariant.

In practice the contribution of the off-shell Born current $J^{B-off}_{\mu\nu}$, which contains the off-shell EM FF, to the amplitude and any observables is proportional to the contraction:

$$T^g(P, q, q') \sim j^{lept}_\alpha(q)\, D^{(g)}_{\alpha\mu}(q^2)\, J^{B-off}_{\mu\nu}(P, q, q')\epsilon_\nu(q'), \quad (7)$$

where $D^{(g)}_{\alpha\mu}(q^2)$ is the propagator of the virtual photon and index g denotes its gauge; $\epsilon_\nu(q')$ is the polarization vector of the real photon, and $j^{lept}_\alpha(q)$ is a leptonic current which corresponds to the $e^-\gamma^\star e^-$ ($\gamma^\star e^+e^-$) vertex for space-like (time-like) photon, and which is conserved automatically (one-photon approximation)

$$q_\alpha\, j^{lept}_\alpha(q) = 0, \quad (8)$$

since the initial/final leptons are on-mass shell.

The gauge transformation of the polarization vector of the real photon

$$\epsilon_\nu(q') \to \epsilon'_\nu(q') = \epsilon_\nu(q') + \lambda q'_\nu \; ; \qquad \lambda \neq 0$$

leads to additional "non-zero" term (proportional to λ) in the amplitude

$$T'^g(P, q, q') = T^g(P, q, q') + \lambda j^{lept}_\alpha(q)\, D^{(g)}_{\alpha\mu}(q^2)\, J^{B-off}_{\mu\nu}(P, q, q')q'_\nu,$$

since current $J^{B-off}_{\mu\nu}(P, q, q')$ is not conserved itself. This shows us directly that the contribution of the off-shell Born curent depends upon the gauge of EM field, and therefore the definition of the off-mass-shell EM form factors is not gauge invariant even if 3-point EM Green functions satisfy corresponding WTI. In the case of off-shell EM currents we have to satisfy not only WTI for 3-points EM Green function, but also all possible high-order WTI for full process (from which this off-shellness appeared) should be satisfyed too. In the same manner it is very easy to see that the *on-shell* EM form factors are defined in a gauge invariant way, since on-shell Born current is conserved itself.

Let us consider the sensitivity of the same amplitude $T^g(P, q, q')$, but with more "general" off-shell Born current $\tilde{J}^{B-off}_{\mu\nu}$ when both (real and virtual) photons' vertices contain the off-shellness and, as a result, $q_\mu \tilde{J}^{B-off}_{\mu\nu} \neq 0$ (see, for instance, representation of the off-shell Born current $\tilde{J}^{B-off}_{\mu\nu}$ in (24) in terms of the *ir-reducible* vertices), to the choice of the gauge for the virtual photon propagator. In general, any known gauge can be used for $D^{(g)}_{\alpha\mu}(q^2)$, but here we will consider only few the most common of them, namely the covariant Feynman ($\eta = 1$) and Landau ($\eta = 0$) gauge

$$D^{(g)}_{\alpha\mu}(q^2) = \frac{1}{q^2 + i0}\{g_{\alpha\mu} + (1-\eta)\frac{q_\alpha q_\mu}{q^2}\}, \qquad (9)$$

and "axial gauge"

$$D^{(g)}_{\alpha\mu}(q^2) = \frac{1}{q^2 + i0}\{g_{\alpha\mu} + \frac{q_\alpha q_\mu}{(nq)^2}n^2 - \frac{n_\alpha q_\mu + q_\alpha n_\mu}{(nq)}\}, \qquad (10)$$

where $n^2 = \pm 1, 0$ corresponds to "axial" ($A_3 = 0$), Weyl ($A_0 = 0$) and "light-cone" ($A_0 - A_3 = 0$) gauges. Substituting (9) into (7) and taking into account (8), we see that the contribution of the $\tilde{J}^{B-off}_{\mu\nu}$ to the amplitude $T^g(P, q, q')$ has a *fixed value* which is the same for both covariant gauges (Feynman and Landau), since the leptonic current is conserved

$$T^{\eta=0}(P, q, q') = T^{\eta=1}(P, q, q').$$

However, in general the conservation of only the leptonic current in (7) is not enough for a gauge invariant definition of the off-shell EM FF. Indeed, substituting (10) into (7) and taking into account that $n_\alpha j^{lept}_\alpha \neq 0$, we see that for any "axial" gauge the amplitude $T^g(P, q, q')$ differs from the "covariant" ones. Moreover, various "axial" gauges, which are *equivalent* in principle, will also lead to different results

$$T^{n^2=1}(P, q, q') \neq T^{n^2=-1}(P, q, q') \neq T^{n^2=0}(P, q, q').$$

Therefore, the definition of the off-shell EM FF in the one-body current, even through $\gamma^\star NN$ vertex which satisfy the WTI for the three-point EM Green function, is gauge dependent and such form factors cannot be investigated in practice. Only exactly conserved hadron currents, like $J^N_{\mu\nu}$ from (6), may lead to *physical* contributions to *observables*.

3 Gauge invariant decomposition of the total nucleon current

The reducible half-off-shell $\gamma^\star NN$- vertex, satisfying the WTI (3), contains two terms [1] $\Gamma^\pm_\mu(q^2, p^2 \neq M^2)$ corresponding to the positive and negative energy states of the virtual nucleon

$$\Gamma_\mu = \Gamma^+_\mu(q^2, p^2)\,\frac{\not p + M}{2M} + \Gamma^-_\mu(q^2, p^2)\,\frac{-\not p + M}{2M}, \qquad (11)$$

which can be expressed [1-3] through the *off-shell* EM FF $F^\pm_{1,2}(q^2, p^2)$:

$$\Gamma^\pm_\mu(q^2, p^2) = e\Big\{F^\pm_1(q^2, p^2)\gamma_\mu + \frac{1 - F^\pm_1(q^2, p^2)}{q^2}\not q q_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2M}F^\pm_2(q^2, p^2)\Big\}, \qquad (12)$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and $\not q = \gamma_\mu q_\mu$.

For an on-mass-shell particle only $\Gamma^+_\mu(q^2, p^2 = M^2)$ exists, and $F^+_{1,2}(q^2, p^2)$ may be considered as the *physical* FF of the nucleon, contrary to $F^-_{1,2}(q^2, p^2)$.

For simplicity we will not consider the Pauli FF $F^\pm_2$ in (12) explicitly. However, we stress that the procedure, similar to that one described below, may be directly applyed to the full vertex in (12).

At the photon point gauge invariance demands [2,10] $F^\pm_1(q^2 = 0, p^2) \equiv 1$. Therefore, assuming a regular behaviour of the *physical* FF as a function of q^2 and p^2 (which is evident below the pion threshold), we get a model independent representation

$$F^+_1(q^2, p^2) = F^+_1(q^2, M^2) + \frac{q^2}{M^2}\frac{p^2 - M^2}{M^2}A^+(q^2, p^2). \qquad (13)$$

Here the analytical function $A^+(q^2, p^2)$ contains all off-shell effects

$$A^+(q^2, p^2) = \sum_{i,k=1}^{\infty} a^+_{ik}\,(\frac{p^2 - M^2}{M^2})^{i-1}(\frac{q^2}{M^2})^{k-1}, \qquad (14)$$

and a^+_{ik} are constants, corresponding to the i, k- order derivatives at $p^2 = M^2$, $q^2 = 0$.

To derive a relation between *off*- and *on-shell* Born currents, we substitute (11,12) into (4). Using (13) and taking into account [6,9] that the numerator $p^2 - M^2$ cancels the denominators of the propagators in (4), as well as the identities: $(\not p + M)S_0(p) = 1 + 2MS_0(p)$ and $(-\not p + M)S_0(p) = -1$, we express $J^{B-off}_{\mu\nu}$ from (4) in terms of the gauge invariant *on-shell* Born current $J^{B-on}_{\mu\nu}$ (containing only *pole-type* singularities), and a regular term (containing only *one-body-irreducible* contributions); the latter consists of a gauge invariant part, $J^{GI}_{\mu\nu}$, and a non-gauge invariant one, $J^{non-GI}_{\mu\nu}$

$$J^{B-off}_{\mu\nu} = J^{B-on}_{\mu\nu} + J^{GI}_{\mu\nu} + J^{non-GI}_{\mu\nu}. \qquad (15)$$

The explicit form of the various currents in (15) is

$$J^{B-on}_{\mu\nu} = e^2\{\Gamma^{on}_\mu S_0(p' + q')\gamma_\nu + \gamma_\nu S_0(p - q')\Gamma^{on}_\mu\}. \qquad (16)$$

$$J^{GI}_{\mu\nu} = \frac{e^2q^2}{M^4}\{A^+(q^2, W^2)\tilde\gamma_\mu \not q'\gamma_\nu - \gamma_\nu \not q'\tilde\gamma_\mu A^+(q^2, W'^2)\}. \qquad (17)$$

$$J^{non-GI}_{\mu\nu} = \frac{2e^2q^2}{M^4}\{p'_\nu A^+(q^2, W^2) + p_\nu A^+(q^2, W'^2)\}\tilde\gamma_\mu + \frac{e^2}{2M}\{\Delta(W^2)\tilde\gamma_\mu\gamma_\nu + \gamma_\nu\tilde\gamma_\mu\Delta(W'^2)\}. \qquad (18)$$

Here $\Delta(W^2) = F^+_1(q^2, W^2) - F^-_1(q^2, W^2)$, $W^2 = (p' + q')^2$, $W'^2 = (p - q')^2$, and

$$\Gamma^{on}_\mu = \Gamma^+_\mu(q^2, M^2); \qquad \tilde\gamma_\mu = \gamma_\mu - \frac{\not q q_\mu}{q^2} \qquad (19)$$

From (15) we see that *off-shell effects* in the $\gamma^\star NN$-vertex may be completely moved (see also [6-9]) to the regular piece of the amplitude (17,18), containing only irreducible graphs. This statement is general and does not depend upon the type of the EM process (it may be applied directly to the photo/electro- disintegration of a bound system).

Using (16-18), it is easy to see that every current in (15) has a non-singular limit at $q^2 \to 0$, and in the photon point (real Compton scattering off a Dirac or scalar particle) off-shell effects vanish (as a consequence of gauge invariance)

$$J^{B-off}_{\mu\nu}(q^2=0) \equiv J^{B-on}_{\mu\nu}(q^2=0)$$

$$J^{GI}_{\mu\nu}(q^2=0) \;=\; J^{non-GI}_{\mu\nu}(q^2=0) \;=\; 0. \qquad (20)$$

To calculate the contribution of the "minimal" contact current we substitute (11) into (5). Then, taking into account requirements of covariance, the integration in (5) can be performed analytically, independent of the explicit form of the vertex function

$$\begin{aligned} -\frac{1}{e}J^C_{\mu\nu} &= [\Gamma^+_\mu(W^2) - \Gamma^-_\mu(W^2)]\gamma_\nu \\ &+\gamma_\nu[\Gamma^+_\mu(W'^2) - \Gamma^-_\mu(W'^2)] \\ &+ \frac{p'_\nu}{p'q'}[\Gamma^+_\mu(W^2) - \Gamma^+_\mu(M^2)] \\ &-\frac{p_\nu}{pq'}[\Gamma^+_\mu(W'^2) - \Gamma^+_\mu(M^2)]. \qquad (21) \end{aligned}$$

Substituting (12,13) into (21), we find that the contact current (21) exactly coincides with the non-gauge invariant piece of the regular term (18), but with opposite sign

$$J^C_{\mu\nu} = -J^{non-GI}_{\mu\nu}. \qquad (22)$$

Summing (22) and (15), we see that the "minimal" contact current (22) kills all non-gauge invariant terms in $J^{B-off}_{\mu\nu}$, and as a result, off-shell effects are completely cancelled in the "longitudinal" part (in the charge operators) of the total conserved nucleon current $J^N_{\mu\nu}$

$$J^N_{\mu\nu} = J^{B-off}_{\mu\nu} + J^C_{\mu\nu} = J^{B-on}_{\mu\nu} + J^{GI}_{\mu\nu}, \qquad (23)$$

where $q_\mu J^{B-on}_{\mu\nu} = J^{B-on}_{\mu\nu}q'_\nu = q_\mu J^{GI}_{\mu\nu} = J^{GI}_{\mu\nu}q'_\nu = 0$, and only the *new* "transverse" contact current $J^{GI}_{\mu\nu} \sim \tilde{\gamma}_\mu\sigma_{\nu\alpha}q'_\alpha$, caused by the Dirac magnetic moment, depends on off-shell effects. In case of VCS off a scalar particle the cancellation of off-shell effects is complete. As it may be seen from (17,18), the expansion (14) was not used to derive (23).

Therefore, we obtained a novel decomposition (23) of the total conserved nucleon current into two gauge invariant pieces $J^{B-on}_{\mu\nu}$ and $J^{GI}_{\mu\nu}$ which shows: 1) the introduction of *off-shell* FF in the "reducible" $\gamma^\star NN$ vertex is not gauge invariant and depends on the representation of the total conserved current; 2) off-shell effects may be distributed over the $\gamma^\star NN$- vertex and the contact current in different ways; 3) off-shell effects from one-body currents are completely cancelled in the longitudinal components of the total conserved current by the "minimal" contact current; 4) F_1^- is removed from the total conserved nucleon current and does not influence the observables. So, even if the gauge constraint for 3-point EM Green functions allows the off-shell effects in the charge (Dirac) coupling (reducible vertex with virtual photon), the high-order gauge constraint for 4-point Green function "kiles" them in the total VCS amplitude. Therefore, in addition we have shown that a representation of the total conserved current such that the off-shell effects enter only in the *non-pole* gauge invariant piece (as it was supposed in [8,9,11] from the beginning) and only through the Dirac magnetic operators exists for both space- and time-like regions. This enables one to introduce an "on-shell extrapolated" FF for the "unphysical" region in a gauge invariant way.

Note, that inclusion of the $F_2^\pm$ form factors in the same manner does not change the structure and the physical meaning of (23). This leads just to only an additional term in $J^{GI}_{\mu\nu}$, containing the physical FF F_2^+ only, proportional to:

$$\left(\frac{d}{dp^2}F_2^+(q^2,p^2)\right)_{p^2=M^2}\{\sigma_{\mu\alpha}\sigma_{\beta\nu} + \sigma_{\beta\nu}\sigma_{\mu\alpha}\}q_\alpha q'_\beta.$$

There are also two additional terms in $J^{non-GI}_{\mu\nu}$, including both $F_2^\pm$ form factors; we do not present these terms here, since they are cancelled by the corresponding terms from the "minimal" contact current, in the same manner as was shown above for $F_1^\pm$. Therefore, we can conclude, that the form factor F_2^- is also removed from the total conserved current, similary to F_1^-, and does not influence the observables.

We stress that the real Compton scattering amplitude off a Dirac (or scalar) particle cannot contain any off-shell effects at all, as a consequence of gauge invariance.

Finally, all results and conclusions, obtained above, are valid for both space/time-like regions, and the definition of the *on-shell Born* current is *unique* under the condition that corresponding $\gamma^\star NN$ vertices satisfy the WTI on the *operator level* (this is always true for a *Dirac* coupling).

4 Conserved current in terms of irreducible vertices

Here we will briefly repeat the same procedure of the "minimal substitution" of the EM field in the *left-hand* side of (1), which contains full propagators and an irreducible vertex. In this case minimal insertion of the second (real) photon into external lines (full renormalized nucleon propagators) generates [10], similar to (4), an alternative expression for the one-body *off-shell* Born current

$$\begin{aligned} \tilde{J}^{B-off}_{\mu\nu} &= \Gamma^{\rm ir}_\mu(p,p'+q')S(p'+q')\Gamma^{ir}_\nu(p'+q',p') \\ &+\Gamma^{ir}_\nu(p,p-q')S(p-q')\Gamma^{\rm ir}_\mu(p-q',p'). \quad (24) \end{aligned}$$

Note, (24) is formally the same as the one used in ref [9] for the contribution of the "class A" diagrams. However, in this paper the *irreducible* γNN vertex (with a real photon) is now the "minimal" three-point EM Green's function connected with inverse propagator [10] (see also [13])

$$\left\{ \begin{array}{l} \Gamma_\nu^{ir}(p'+q',p') \\ \Gamma_\nu^{\rm ir}(p,p-q') \end{array} \right\} = \pm e \int_0^1 \frac{d\lambda}{\lambda} \frac{d}{dq'_\nu} \left\{ \begin{array}{l} S^{-1}(p'+\lambda q') \\ S^{-1}(p-\lambda q') \end{array} \right\}, \quad (25)$$

and, as a result, satisfies the WTI (2) automatically. Equation (25) is a consequence [10] of the "minimal coupling" of the second (real) photon to the full renormalized propagator, and follows from the (21)-(22) in the first [10]. The *irreducible* $\gamma^\star NN$ vertices (with *virtual* photons), $\Gamma_\mu^{\rm ir}(p'+q',p)$ and $\Gamma_\mu^{\rm ir}(p-q',p')$, satisfy the WTI (2) also [12]. However, for these we do not need a "minimal" procedure, and their Lorentz-spin structure may be described by the same equations like (11, 12).

The minimal photon insertion inside the *irreducible* $\gamma^\star NN$ vertex gives rise to the new contact current which has the same structure as (5); however it contains *irreducible* vertices instead of *reducible* ones

$$\tilde{J}^C_{\mu\nu} = -e \int_0^1 \frac{d\lambda}{\lambda} \frac{d}{dq'_\nu} \{ \Gamma_\mu^{\rm ir}(p,p'+\lambda q') - \Gamma_\mu^{\rm ir}(p-\lambda q',p') \}. \quad (26)$$

Using the WTI (2) and (25), as well as taking into account that $\bar{u}(p)S^{-1}(p) = \bar{u}(p)S_0^{-1}(p) = S^{-1}(p')u(p') = S_0^{-1}(p')u(p') = 0$, it is easy to get convinced that the derivatives of $\tilde{J}^C_{\mu\nu}$ cancel corresponding derivatives of $\tilde{J}^{B-off}_{\mu\nu}$, independent of the explicit form of the vertices, and the *new* total nucleon current

$$\tilde{J}^N_{\mu\nu} \;\; = \;\; \tilde{J}^{B-off}_{\mu\nu} \;\; + \;\; \tilde{J}^C_{\mu\nu} \; , \quad (27)$$

defined in the terms of the *irreducible* vertices and *full* renormalized propagators, is exactly conserved

$$q_\mu \tilde{J}^N_{\mu\nu} \;\; = \;\; \tilde{J}^N_{\mu\nu} q'_\nu \;\; = \;\; 0.$$

To establish a relation between two gauge invariant nucleon currents, $J^N_{\mu\nu}$ from (6) and $\tilde{J}^N_{\mu\nu}$ from (27), we have to take into account the connection between "full" renormalized and "free" Feynman propagators

$$\begin{array}{l} S^{-1}(p) = \not{p} - M - \Sigma(p); \;\; S_0^{-1} = \not{p} - M; \\ S_0^{-1}(p) - S^{-1}(p) = \Sigma(p), \end{array} \quad (28)$$

where the mass-operator $\Sigma(p)$ satisfies the following conditions:

$$\Sigma(p)|_{p^2=M^2} = 0 \quad ; \quad \frac{d}{d\not{p}} \Sigma(p)|_{p^2=M^2} = 0. \quad (29)$$

Substitution of (28) into (25) gives the connection between the "minimal" γNN *irreducible* vertex and the *reducible* one

$$\Gamma_\nu^{\rm ir}(p'+q',p') = e\gamma_\nu - e \int_0^1 \frac{d\lambda}{\lambda} \frac{d}{dq'_\nu} \Sigma(p'+\lambda q'), \quad (30)$$

Using (30) and taking into account that $\Gamma_\mu^{\rm ir}(p,p'+q')S(p'+q') = \Gamma_\mu(p,p'+q')S_0(p'+q')$ and $S(p-q')\Gamma_\mu^{\rm ir}(p-q',p') = S_0(p-q')\Gamma_\mu(p-q',p')$, we can identify in (24) the piece that contains only *reducible* vertices and *free* Feynman propagators, i.e. the part which exactly coincides with $J^{B-off}_{\mu\nu}$ from (4), and a correction term coming from the mass-operator (see Appendix D):

$$\tilde{J}^{B-off}_{\mu\nu} \;\; = \;\; J^{B-off}_{\mu\nu} \;\; + \;\; \delta J^{B-off}_{\mu\nu}, \quad (31)$$

The same procedure for the contact current $\tilde{J}^C_{\mu\nu}$ (26) (see Appendix D) allows us to identify the piece which contains only *reducible* vertices and, of course, exactly coincides with our old contact current $J^C_{\mu\nu}$ from (5) plus correction term $\delta J^C_{\mu\nu}$ caused also by the mass-operator:

$$\tilde{J}^C_{\mu\nu} \;\; = \;\; J^C_{\mu\nu} \;\; + \;\; \delta J^C_{\mu\nu}, \quad (32)$$

The direct calculation of $\delta J^C_{\mu\nu}$ gives (see Appendix D):

$$\delta J^C_{\mu\nu} \;\; = \;\; -\delta J^{B-off}_{\mu\nu}. \quad (33)$$

Thus, substituting (31,32) into (27) and taking into account (33) we conclude

$$\tilde{J}^N_{\mu\nu} \;\; \equiv \;\; J^N_{\mu\nu}. \quad (34)$$

Therefore, in the framework of the "minimal" scheme for the gauge field the corrections, connected with the mass-operator, to the off-shell Born current and to the contact current cancel one another in the total conserved current. As a result, its definition in terms of the irreducible vertices and full renormalized propagators as well as in terms of the reducible vertices and free Feynman propagators is *equivalent*.

Finally, (34) is also satisfied if both photons are real (this can be shown by using the above described procedure with additional symmetrization [14] in (26) on photon lines). Note, the identity (34) leads to some restrictions for the off-shell effects at the "photon-point", as a consequence of the gauge invariance. Indeed, in accordance with (20), the right hand side of (34), i.e. current $J^N_{\mu\nu}$, does not contain any off-shellness for the real Compton scattering on a Dirac (or scalar) particle. This means that the off-shell effects in the left hand side of (34), i.e. in the current $\tilde{J}^N_{\mu\nu}$, caused by the self-energy part, appear in the correction terms only, and cancel in the total gauge invariant amplitude in a model independent way.

5 Illustration: time-like VCS in "unphysical" region

As an illustration we consider the possibility to introduce an on-shell extrapolated Dirac FF in "unphysical" region and to investigate it experimentally in the di-lepton photoproduction off a proton. All information about time-like FF is concentrated in the VCS amplitude which is much smaller than Bethe-Heitler (BH) one in general. Therefore, their separation by measuring the C- odd e^+e^-- asymmetry in the $\gamma p \to p e^+ e^-$ reaction

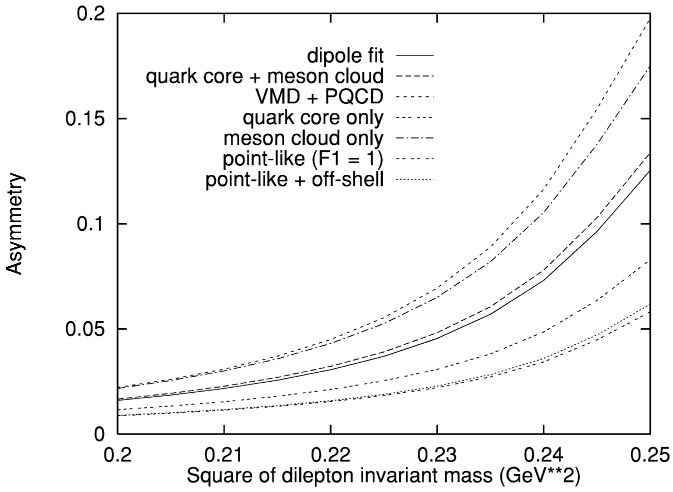

Fig. 1. Predictions for the asymmetry $A_{e^+e^-}$ for different models for $F_1(q^2, M^2)$

has been proposed [14]. The selection of "longitudinal" photons $\mathbf{q}^2 \ll q^2 \ll 4M^2$ in the collinear kinematics provides [14] a strong suppression of the Pauli FF F_2 and the contribution from resonances, which are excited mainly by magnetic (transverse) transitions. As a result, for these "longitudinal" photons the *physical* proton resembles a Dirac one [14]. Taking into account that squares of BH/VCS amplitudes are C- even under permutation of e^+e^- ($T^{BH,VCS}_{-+} = T^{BH,VCS}_{+-}$), while their interference is C- odd ($T^{int}_{-+} = -T^{int}_{+-}$), one can introduce the e^+e^--asymmetry [14] ($K^{-1}_{-+} = E_- + E_+ \cos(\theta) + p_0$):

$$A_{e^+e^-} = \frac{K^{-1}_{+-}\sigma_{+-} - K^{-1}_{-+}\sigma_{-+}}{K^{-1}_{+-}\sigma_{+-} + K^{-1}_{-+}\sigma_{-+}} \equiv \frac{T^{int}_{-+}}{T^{BH}_{-+} + T^{VCS}_{-+}} \approx T^{int}_{-+} \,/\, T^{BH}_{-+}, \qquad (35)$$

where $E_\pm$ and θ are the energy of positron/electron and the angle between them. We use c.m. system with z- axis fixed along 3-momentum of the virtual photon $\mathbf{q}$, and $\sigma_{+-} = d^5\sigma_{+-}/dE_- d\Omega_- d\Omega_+$.

The $A_{e^+e^-}$- asymmetry is shown in Fig. 1 as a function of q^2 at $E_\gamma = 0.65$ GeV, $\theta = 165^o$, $\theta_{\gamma\gamma^\star} = 0^o$ (collinear kinematics with almost "longitudinal" photons: $\mathbf{q}^2 \ll q^2$). To get an idea about the sensitivity of asymmetry to the q^2- dependence, we compare several models for $F_1(q^2)$ assuming the vector meson dominance (VMD). The chiral quark-bag model [15], containing the coupling of the photon to the meson cloud (VMD piece) and directly to the quark core, dispersion relation calculations [16], including constraints from the hadron-channel unitarity (part corresponding to VMD) and perturbative QCD, as well as the well known "dipole fit" were extrapolated to the time-like "unphysical" region. All of them practically coincide and fit experimental data well in the space-like region. When extended into the time-like sector, they give rise to a resonance-like peak corresponding to the VMD contribution [4], and different results even at small positive q^2. We have also calculated the asymmetry for the "point-like" proton, when $F_1(q^2) = 1$.

It is interesting to observe that due to the $\pi\pi$- continuum contribution to the isovector spectral function [16], dispersion relation calculations (VMD + PQCD) predict practically the same asymmetry as a pure meson cloud piece of the chiral quark-bag model, while the full quark-bag model gives the result which is very close to the dipole fit prediction. The comparison of these models with the "point-like" proton prediction shows a strong sensitivity of the asymmetry to the EM structure of the proton. All curves in Fig. 1 (except the dotted curve) were obtained without off-shell effects: $F_1^+(q^2, W^2) \to F_1^+(q^2, M^2)$, and expansion (14) was not used at all.

To estimate the size of off-shell effects (which come only from $J^{GI}_{\mu\nu}$ in a new representation (23) of the full conserved current) we took into account the leading order in the expansion (13) on the hadron virtuality: $a_{11} \neq 0$ and $a_{ik} = 0$, if $i, k > 1$. A small difference between two lowest curves in the picture, dotted and double-dashed ones, obtained for the "point-like" proton, but with $a_{11} = 1$ and 0, respectively, reflects the insignificant role of off-shell effects in the full conserved current (6). Therefore, selection of "longitudinal" photons [14] in collinear kinematics of $\gamma p \to p e^+ e^-$ reaction can provide a practically model-independent investigation of the "on-shell extrapolated" Dirac FF in the "unphysical" region.

6 Discussion and conclusions

In this paper "off-shell Born current" (irregular piece of the amplitude) and "contact current" (regular piece of the amplitude) were obtained "simultaneously" from the same principle in an explicit form, using only gauge properties of the EM field, in terms of the well-known structure [1-4] of the half-off-shell vertex $\gamma^\star NN$. So, a completely consistent treatment of irregular and regular pieces of the total VCS amplitude off a Dirac proton was achieved without assuming a dynamical model for off-$shellness$. This allows us to consider the cancellation of the off-shell effects inside the total gauge invariant current in a model independent way. Two different representations of the "minimal" conserved current in terms of the *reducible* vertices and $free$ Feynman propagators, as well as *irreducible* vertices and $full$ renormalized propagators were considered and their equivalence was shown.

To summarize, the definition of the off-shell EM FF through reducible or irreducible $\gamma^\star NN$ vertices is not gauge invariant even if these vertices satisfy WTI for the 3-point EM Green function. For the gauge invariant definition of the (half)-off-shell form factors in the 3-point EM vertex with even one off-mass-shell hadron line it is **not sufficient** to satisfy WTI for the 3-point EM Green function only. Such an off-shell vertex does not exist independently, it is only a part of more complicated current (diagram) which should satisfy next (high)-order WTI. So far, for a gauge invariant definition of the off-shell EM form factors all high-order WTI (for 4-, 5-, ... point EM

Green functions) for the **full diagrams** (in which that off-shell 3-point EM vertex appears) should be satisfied. So, on the one hand, such an individual property of a particle as the form factor may be defined through the "one-body" current only. However, on the other hand, a "one-body" current is not conserved itself if it contains a coupling of the photon to the virtual hadrons, and a form factor defined in such a way will depend upon the gauge. On the contrary, to satisfy higher-order WTI (for the full amplitude) a "contact current" should be included into the definition of the off-shell form factors. However, in this case off-shellness in the one-body off-shell current may be cancelled by the off-shell effects coming from the contact current, like in the VCS, and the total amplitude may be free from off-shell effects at all. In general, any "dressing" of the one-body properties (such as form factors) by the "many-body" effects is not unique and cannot be observed in practice.

For the pure EM processes, like VCS, off-shell effects in the *reducible* $\gamma^\star NN$ vertices may not only be moved to the regular part of the amplitude (analogously to the result [8,9] for *ir-reducible* vertices), but, and it is more important, they are completely **cancelled** in the longitudinal components of the total conserved current by the "minimal" contact current, when pole and contact amplitudes are defined consistently. Therefore, charge (Dirac) operators are completely free from off-shellness due to gauge constraint. This means that indeed such a model independent representation of the total conserved current, when off-shell effects enter only in the transverse (magnetic) gauge invariant *non-pole* part, exists for both space/time-like regions. This enables one to introduce an "on-shell extrapolated" FF for the "unphysical" region in a gauge invariant way which could be investigated in practice. The *unphysical* FF $F^-_{1,2}$ is removed from the full amplitude due to current conservation and does not influence the observables.

The off-shellness in the *irreducible* vertices, which appears even at photon point and which is caused by the mass-operator corrections, is cancelled in the "minimal" conserved current in a model independent way, since self-energy corrections to the Born and contact currents are the same, but they have opposite signs. In general this means that the requirements of gauge invariance impose model independent restrictions on the off-shell properties of the charge operators at the "photon point". As a result, any off-shell effects cannot exist in the real Compton scattering amplitude off a scalar (or Dirac) particle as a consequence of a gauge invariance only. Indeed, in this case any off-shellness which appear in the "irreducible representation" may be cancelled simply by reformulating the theory in terms of the *reducible* vertices and *free* Feynman propagators.

Taking into account both results for the reducible and irreducible vertices, for both real and virtual photons, we can conclude: even if the gauge constraint for 3-point EM Green functions allows for the hadronic off-shellness in the charge (Dirac) form factors, it vanishes in the total Real/Virtual Compton scattering amplitude due to gauge constraint for 4-point EM Green function only.

The above mentioned results were obtained in the framework of the "minimal" gauge scheme without model suppositions about off-shell behaviour of the nucleon form factors. Of course, such a consideration is restricted to only the (charge) Dirac γNN coupling, associated with *real photon*, since gauge constraints on the level of "tree"-type diagram do not affect a "transverse" part of the current. The last one cannot be obtained on the "tree"- level through the "minimal photon insertions" into the 2- and 3-point Green functions only [10]. This was already mentioned in the deutron photodisintegration [10], and also pion photoproduction [17,18]. In the last case it was shown that so called "Ohta prescription [13]", which involves the "minimal substitution" on the "tree"-level only, is *not complete*, since does not generate full current structure, in particular, its "purely transverse" part [19]. To generate a "transverse" part the "minimal substitution" on the level of the loop-diagrams corresponding to the next order "vertex corrections" should be considered (see second ref. in [10]). Indeed, additional "photon insertions" into the all internal (charged) lines and vertices, i.e. into the 2-, 3- and 4-point Green functions belonging to the loop-diagrams, will reproduce an additional "transverse" part [10] (in the same way like in the QED [12]). Therefore, additional gauge constraints for 3-, 4- and 5-point EM Green functions should be considered to examine off-shell effects in the "transverse" part. This evidently requires a knowledge of the strong interaction structure, and one can expect the evaluation of the off-shellness in the "transverse" part, in respect to the *real photon*, may be model dependent (see, for instance, [20-22]). However, this is beyond of our consideration in this paper. As for the $\gamma^\star NN$ vertices, associated with *virtual photons*, in general, both Dirac and Pauli couplings may formally be included, since we do not suppose "minimal" procedure in this case and we can use the most general phenomenological expression satisfying fundamental requirements. Therefore, the off-shellness arising from the "magnetic" coupling in the $\gamma^\star NN$ vertex (see (12)) may also be moved to the contact current in a manner similar to (13), (15)-(18).

Finally, measurements of $A_{e^+e^-}$- asymmetry in the $\gamma p \to pe^+e^-$ reaction with "longitudinal" photons, with a mass much more larger than 3-momentum, can be used to investigate the on-shell extrapolated Dirac FF $F_1^+(q^2, M^2)$ of the proton in the time-like region $4m_e^2 < q^2 < 4M^2$.

When this work was completed, some attempt to estimate the role of off-shell effects in the VCS was published [23]. Also, the cancellation of the off-shell effects in the bremsstrahglung amplitude was discussed in the frameworks of the chiral perturbative theory [24], however, accounting for the "non-perturbative" effects in the hadronic T- matrix (bound states) may change such a conclusion considerably.

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Appendix A

The general form of the Ward-Takahashi identity for the *reducible* three-point Green function with the EM field (vertex γNN in our case) is well known

$$q_\mu \Gamma_\mu(p,p') = eS_0^{-1}(p)[S(p') - S(p)]S_0^{-1}(p'). \qquad (A.1)$$

Using the asymptotic relation $\lim_{p^2\to M^2} S_0^{-1}(p)S(p) = 1$ the gauge constraint for the *half-off-shell* reducible vertex becomes (let us put p is an on-shell momentum, while p' is an off-shell one: $p^2 = M^2$, $p'^2 \neq M^2$, and $u(p)$ is a Dirac bi-spinor, i.e. a wave function of the on-shell particle):

$$\bar{u}(p)q_\mu \Gamma_\mu(p,p') = e\bar{u}(p)S_0^{-1}(p)[S(p') - S(p)]S_0^{-1}(p') = e\bar{u}(p)\{S_0^{-1}(p) - S_0^{-1}(p')\} \qquad (A.2)$$

Therefore from (A.1) and (A.2) we see that the simple form of (3) for the Ward-Takahashi identity may be used for the half-off-shell γNN vertex.

Appendix B

The structure of the VCS *off-shell Born* current in (4) will be considered here on the basis of the “minimal insertion” of the EM field. We start with the definition of the reducible $\gamma^\star NN$ vertex $\Gamma_\mu(p',p)$ in terms of the irreducible $\Gamma_\mu^{\rm ir}(p',p)$ one, (1).

Since the *reducible* vertex in (1) is surrounded by free propagators, we have to deal with the “minimal insertion” of the external EM field $A_\nu(q')$, corresponding to the real (second) photon, into the “free” external hadron lines in the right-hand side of (1)

$$S_0(p')\Gamma_\mu(p',p)S_0(p) \to S_0(p'-eA)\Gamma_\mu(p',p)S_0(p-eA) \equiv G_\mu(p',p,\{A\}) \qquad (B.1)$$

Then, the corresponding EM current is:

$$J_{\mu\nu}^{B-off} = S_0^{-1}(p')[\frac{\delta}{\delta A_\nu}G_\mu(p',p,\{A\})]_{A\to 0}S_0^{-1}(p), \qquad (B.2)$$

Using (B.1) and (B.2) we obtain the off-shell Born current for VCS in the form of (4).

The same result may be obtained if one takes into account that the “minimal” three-point Green function, corresponding to the “minimal” coupling of the real photon to the external hadron lines, is connected with the corresponding inverse propagator [10] (in our case, see (1), it is the *free* propagator):

$$\Gamma_\nu(p-q',p) = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq_\nu}S_0^{-1}(p-\lambda q') = e\gamma_\nu,$$

Appendix C

A general Lorentz covariant, crossing symmetry, CPT and gauge invariant expression of the *off-shell* nucleon current for the space/time-like VCS, containing independently crossing symmetry off-shell Born term and contact current both in an explicite form as a function of only half-off-shell nucleon form factors in the $\gamma^\star NN$ vertex, was obtained recently [14].

In the particular case, when one photon is *real* and the other *virtual*, the crossing transformation connects the amplitudes of the *time-like* and the *space-like* VCS. Indeed from (4,5) a total gauge invariant current for time-like VCS ($q^2 > 0$, $q'^2 = 0$) is:

$$e^{-1}J_{\mu\nu}^N(\gamma N \to N\gamma^\star) = \Gamma_\mu(p,p'+q')S_0(p'+q')\gamma_\nu + \gamma_\nu S_0(p-q')\Gamma_\mu(p-q',p') - \int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p,p'+\lambda q') - \Gamma_\mu(p-\lambda q',p')\}. \qquad (C.1)$$

Equation (C.1) under crossing transformation $q \to -q'$, $q' \to -q$, $\mu \leftrightarrow \nu$ generates a gauge invariant current for space-like VCS ($q^2 = 0$, $q'^2 < 0$):

$$e^{-1}J_{\nu\mu}^N(\gamma^\star N \to N\gamma) = \Gamma_\nu(p,p'-q)S_0(p'-q)\gamma_\mu + \gamma_\mu S_0(p+q)\Gamma_\nu(p+q,p') - \int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\mu}\{\Gamma_\nu(p+\lambda q,p') - \Gamma_\nu(p,p'-\lambda q)\}, \qquad (C.2)$$

Although the currents (C.1) and (C.2) are not the same (since the initial and final photons are different), they are connected by crossing transformations.

At the photon point (both photons are real) due to WTI(3) all reducible vertices $\Gamma_\mu(p,p')$ in (15,5) become simply a constant ($e\gamma_\mu$). This means that for real photons the reducible half-off-shell γpp vertex of the Dirac proton coincides with the on-shell one and does not contain any *off-shellness* (consequence of the gauge constraint [2,10]). As a result, $J_{\mu\nu}^{B-off}(q^2 = q'^2 = 0) = J_{\mu\nu}^{B-on}(q^2 = q'^2 = 0)$, $J_{\mu\nu}^C(q^2 = q'^2 = 0) = 0$, and in the case of two real photons current (23) is crossing symmetry

$$J_{\mu\nu}^N(P,q',q)|_{q^2=q'^2=0} = J_{\nu\mu}^N(P,-q,-q')|_{q^2=q'^2=0}$$

Appendix D

First, we derive the connection between two different off-shell Born currents $\tilde{J}_{\mu\nu}^{B-off}$ and $J_{\mu\nu}^{B-off}$, defined in terms of the *ir-reducible* vertices and *full* renormalized propagators, and in terms of the *reducible* vertices and *free* Feynman propagators, respectively. Starting from (24), and accounting for the definition (1), we can make transition from the irreducible vertices of the virtual photons and full propagators to the reducible ones and free propagators, while for the real photons we have still irreducible vertices:

$$\tilde{J}^{B-off}_{\mu\nu} = \Gamma_\mu(p,p'+q')S_0(p'+q')\Gamma^{ir}_\nu(p'+q',p') + \Gamma^{ir}_\nu(p,p-q')S_0(p-q')\Gamma_\mu(p'+q',p').$$

The substitution of (30) for irreducible γNN vertices allows us to decompose the "new" off-shell Born current (24) into two terms. The first one is the "old" Born current $J^{B-off}_{\mu\nu}$ from (4), while the second one, $\delta J^{B-off}_{\mu\nu}$, is a correction term coming from the mass-operator:

$$\tilde{J}^{B-off}_{\mu\nu} = \Gamma_\mu(p,p'+q')S_0(p'+q')(e\gamma_\nu) + (e\gamma_\nu)S_0(p-q')\Gamma_\mu(p-q',p') + \delta J^{B-off}_{\mu\nu}, \quad (D.1)$$

$$\delta J^{B-off}_{\mu\nu} = -e\int_0^1 \frac{d\lambda}{\lambda}\{\Gamma_\mu(p,p'+q')S_0(p'+q') \times [\frac{d}{dq'_\nu}\Sigma(p'+\lambda q')] -[\frac{d}{dq'_\nu}\Sigma(p-\lambda q')]S_0(p-q')\Gamma_\mu(p-q',p')\}. \quad (D.2)$$

From (D1,2), and using (30), we immediately get the relation between two off-shell Born currents in the form of (31).

Second, we derive the connection between two the different contact currents $J^C_{\mu\nu}$ and $\tilde{J}^C_{\mu\nu}$ which are defined through irreducible and reducible vertices, respectively. Starting from (26), we can rewrite it as

$$\tilde{J}^C_{\mu\nu} = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma^{\rm ir}_\mu(p,p'+\lambda q') \times S(p'+\lambda q')S^{-1}(p'+\lambda q') -S^{-1}(p-\lambda q')S(p-\lambda q')\Gamma^{\rm ir}_\mu(p-\lambda q',p')\}. \quad (D.3)$$

Using the definition (1), and the relation between *free* and *full* propagators (28), we can make the transition from the irreducible vertices to the reducible ones:

$$\tilde{J}^C_{\mu\nu} = -e\int_0^1 \frac{d\lambda}{\lambda}\frac{d}{dq'_\nu}\{\Gamma_\mu(p,p'+\lambda q')S_0(p'+\lambda q') \times[S_0^{-1}(p'+\lambda q')-\Sigma(p'+\lambda q')] -[S_0^{-1}(p-\lambda q')-\Sigma(p-\lambda q') \times S_0(p-\lambda q')\Gamma_\mu(p-\lambda q',p')\}. \quad (D.4)$$

Isolating in (D.4) terms containing only reducible vertices, we see that the "new" contact current (26) may be presented in the form (32), i.e. expressed through the "old" contact current $J^C_{\mu\nu}$, defined by (5), and an additional correction term $\delta J^C_{\mu\nu}$:

$$\delta J^C_{\mu\nu} = e\int_0^1 \frac{d\lambda}{\lambda}\{[\frac{d}{dq'_\nu}\Gamma_\mu(p,p'+\lambda q')]S_0(p')\Sigma(p') +\Gamma_\mu(p,p'+q')[\frac{d}{dq'_\nu}S_0(p'+\lambda q')]\Sigma(p') +\Gamma_\mu(p,p'+q')S_0(p'+q')[\frac{d}{dq'_\nu}\Sigma(p'+\lambda q')] -[\frac{d}{dq'_\nu}\Sigma(p-\lambda q')]S_0(p-q')\Gamma_\mu(p-q',p') -\Sigma(p)[\frac{d}{dq'_\nu}S_0(p-\lambda q')]\Gamma_\mu(p-q',p') -\Sigma(p)S_0(p)[\frac{d}{dq'_\nu}\Gamma_\mu(p-q',p')]\}. \quad (D.5)$$

Then, taking into account (29), and omitting all terms which are proportional to $\Sigma(p)$ and $\Sigma(p')$ since $p^2 = p'^2 = M^2$, we can write down the "contact current correction":

$$\delta J^C_{\mu\nu} = e\int_0^1 \frac{d\lambda}{\lambda}\{\Gamma_\mu(p,p'+q')S_0(p'+q')[\frac{d}{dq'_\nu}\Sigma(p'+\lambda q')] -[\frac{d}{dq'_\nu}\Sigma(p-\lambda q')]S_0(p-q')\Gamma_\mu(p-q',p')\}. \quad (D.6)$$

Comparing (D.2) and (D.6), we immediately get (33).

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